MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2021/22 (T2130)

EMT2066 – INDUSTRIAL MATHEMATICS

(All Sections / Groups)

16 AUGUST 2022

9.00 AM – 11.00 AM

(2 Hours)

INSTRUCTIONS TO STUDENTS:

1. This exam paper consists of **9 pages** (including cover page) with **4** Questions only.
2. Attempt **all** 4 questions. All questions carry equal marks and the distribution of marks for each question is given.
3. Please print all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.
4. Several formulae are provided in the **Appendix** for your reference.

Question 1

(a) Determine the constants a, b, and x_1 so that $\int_0^2 g(x)\,dx = ag(0) + bg(x_1)$ has degree of precision 2. [7.5 marks]

(b) Approximate the integral $\int_0^2 3x^2\,dx$ by using the formula in Question 1(a). Do you think the obtained answer provides an accurate estimation of $\int_0^2 3x^2\,dx$? Justify your answer. [5.5 marks]

(c) Consider the following system of linear equations:

$$\begin{aligned} 12x_1 + 3x_2 - 5x_3 &= 1 \\ -x_1 - 5x_2 - 3x_3 &= -28 \\ 3x_1 + 7x_2 + 13x_3 &= 76 \end{aligned}$$

By using Gauss-Seidel technique, estimate $[x_1, x_2, x_3]$ after two iterations starting from an initial estimate $[x_1, x_2, x_3] = [1, 0, 1]$. [12 marks]

Continued...

Question 2

(a) Table 1 shows the probability for the movement of a particle from one state to another. Assume that the particle's initial position is in State 1.

Table 1: Probability of the movement of a particle.

Current State	Next State			
	State 0	State 1	State 2	State 3
State 0	0	0	1	0
State 1	1	0	0	0
State 2	a	a	0	0
State 3	0.25	b	b	0.25

(i) Find the values of a and b. [3 marks]

(ii) Calculate the probability that the particle will be at state i, i =0,1,2,3 after the first transition, as well as after the second transition. [5 marks]

(iii) Draw the state transition diagram. [2 marks]

(iv) Decompose the states into equivalence classes. [3 marks]

(v) Determine those equivalent classes whether they are recurrent or transient. [2 marks]

(b) Consider a Markov chain with state space {0, 1, 2} and transition probability matrix

$$\begin{bmatrix} 0 & 0.8 & 0.2 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix}.$$

(i) Is the above transition probability matrix aperiodic and irreducible? Justify your answer. [3 marks]

(ii) Calculate the limiting state (stationary) probability vector $\begin{bmatrix} \pi_0 & \pi_1 & \pi_2 \end{bmatrix}$. [7 marks]

Continued...

Question 3

(a) Consider the following linear programming (LP) problem:

$$\begin{aligned} \text{Maximize } z = \; & x_1 - x_2 + 2x_3 \\ \text{subject to:} \quad & 3x_1 - x_2 + x_3 \leq 4 \quad \text{(Constraint 1)} \\ & x_2 + 2x_3 \leq 11 \quad \text{(Constraint 2)} \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

By introducing slack variables where appropriate, express the LP problem in standard form. Then solve the LP problem using the simplex method. Be sure to specify the entering and leaving variables, as well as the elementary row operations for each iteration. For the benefit of comparison, a partially-filled simplex tableau at the optimal stage is shown below:

Basic	z	x_1	x_2	x_3	s_1	s_2	Solution
z	1	3	0	0	4/3	1/3	9
?	0	?	?	?	?	?	5
?	0	?	?	?	?	?	1

[15 marks]

(b) Convert the above LP problem to its dual.

[5 marks]

(c) Using appropriate properties, deduce the optimal solution and optimal value for the dual problem in (b).

[3 marks]

(d) What would happen to the optimal value if:

(i) the right-hand-side (RHS) of Constraint 1 is increased by 1 unit?

(ii) the RHS of Constraint 2 is decreased by 1 unit?

[2 marks]

Continued...

Question 4

(a) **Figure 4a** below shows a computer network consisting of seven terminals, represented by nodes 1 through 7. A branch between two nodes indicates a direct communications link, and the number on each branch is the average time (in milliseconds) to transmit a data packet via that link.

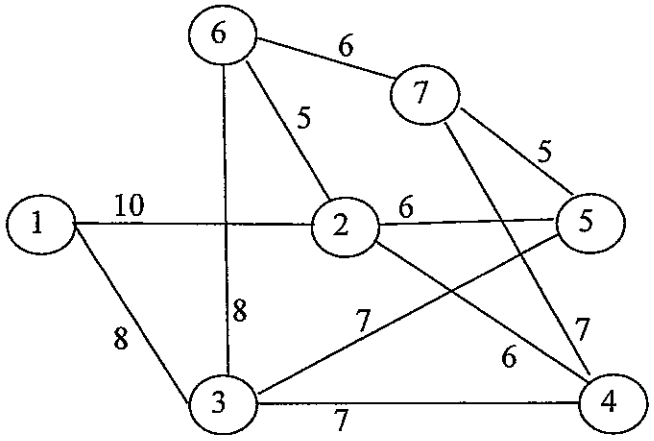

Figure 4a

(i) Suppose a data packet from terminal 1 is to be routed to terminal 7. Using dynamic programming (DP), determine a route that minimizes the packet transmission time. What would this minimum transmission time be? (**Hint**: You are advised to redraw **Figure 4a** to clearly delineate the DP stages.)

[13 marks]

(ii) If terminal 3 malfunctions, which alternative route minimizes transmission time from terminals 1 to 7?

[2 marks]

Continued...

Question 4

(b) A recent firmware update established direct comm. links between terminals 1 and 6, and terminals 4 and 5. **Figure 4b** below incorporates this new information:

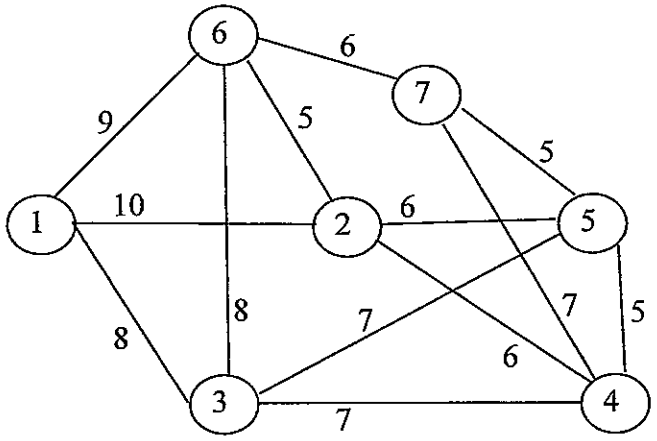

Figure 4b

(i) The network undergoes maintenance every night between 11.30 and 12.30pm. During this period, the functionality of the network is reduced to the bare minimum by operating as a minimum spanning tree. Using Kruskal's algorithm, demonstrate how this minimum spanning tree may be obtained from **Figure 4b**.

[8 marks]

(ii) Security protocol requires that terminal 1 should be connected to at least two other terminals at all times. Draw the minimum spanning tree from **Figure 4b** that satisfies this requirement.

[2 marks]

Continued...

APPENDIX I: List of Formulas

1. The nth Lagrange interpolating polynomial (LIP)

$$f(x) \approx P_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i)$$

with

$$L_k(x) = \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{(x - x_i)}{(x_k - x_i)}.$$

2. Newton's divided-difference interpolating polynomial (NDDIP)

$$P_n(x) = f[x_0] + \sum_{k=1}^{n} f[x_0, x_1, \ldots, x_k](x - x_0) \cdots (x - x_{k-1}).$$

3. The error in interpolating polynomial.

$$f(x) - P_n(x) = \frac{(x - x_0)(x - x_1) \ldots (x - x_n)}{(n+1)!} f^{(n+1)}(c_x)$$

for each $x \in [x_0, x_n]$, a number $c_x \in (x_0, x_n)$ exists.

4. Newton's forward-difference formula

$$P_n(x) = f[x_0] + \sum_{k=1}^{n} \binom{s}{k} \Delta^k f(x_0).$$

5. Newton's backward-difference formula

$$P_n(x) = f[x_n] + \sum_{k=1}^{n} (-1)^k \binom{-s}{k} \nabla^k f(x_n).$$

6. Forward difference formula

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}.$$

Backward difference formula

$$f'(x) \approx \frac{f(x) - f(x-h)}{h}.$$

The error term for both forward and backward difference formula is

$$\left| \frac{h}{2} f''(c_x) \right|.$$

Continued...

7. Central difference formula

$$f'(x) \approx \frac{f(x+h)-f(x-h)}{2h}$$

with the error term

$$\left|\frac{h^2}{6} f^{(3)}(c_x)\right|.$$

8. Trapezoidal rule

$$\int_a^b f(x)dx \approx \frac{h}{2}\left(f(a)+f(b)\right).$$

The error term is $\left|\frac{h^3 f''(\xi)}{12}\right|$ for some ξ in (a, b) and $h = b - a$.

9. Composite Trapezoidal rule

$$\int_a^b f(x)dx \approx \frac{h}{2}\left[f(a)+f(b)+2\sum_{j=1}^{n-1} f(x_j)\right].$$

The error term is $\left|\frac{(b-a)h^2 f''(\xi)}{12}\right|$ for some ξ in (a, b) and $h = \frac{b-a}{n}$.

10. Simpson's rule

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right].$$

The error term is $\left|\frac{h^5}{90} f^{(iv)}(\xi)\right|$ for some ξ in (a, b) and $h = \frac{b-a}{2}$.

11. Composite Simpson's rule

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a)+2\sum_{j=1}^{\left(\frac{n}{2}\right)-1} f(x_{2j})+4\sum_{j=1}^{\frac{n}{2}} f(x_{2j-1})+f(b)\right].$$

The error term is $\left|\frac{(b-a)h^4}{180} f^{(4)}(\xi)\right|$ for some ξ in (a, b), $h = \frac{b-a}{n}$, $x_j = a + jh$, $j = 0, 1, \ldots, n$.

Continued...

12. Four-term Gaussian Quadrature

$$\int_a^b f(x)dx = \frac{(b-a)}{2}\int_{-1}^{1} f\left(x = \frac{(b+a)+t(b-a)}{2}\right)dt$$

$$\int_{-1}^{1} f(x)dx \approx w_1 f(x_1) + w_2 f(x_2) + w_3 f(x_3) + w_4 f(x_4)$$

Number of terms n	Nodes x_i	Weights w_i
4	-0.8611363116	0.3478548451
	-0.3399810436	0.6521451549
	0.3399810436	0.6521451549
	0.8611363116	0.3478548451

13. Newton-Raphson's method

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}, \qquad n = 1,2,...$$

14. Euler's method

$$y_{i+1} = y_i + hf(x_i, y_i)$$

with local error $\frac{h^2}{2}Y''(\xi_i)$ for some ξ_i in (x_i, x_{i+1}).

15. Runge Kutta method of order two (Improved Euler method)

$$y_{i+1} = y_i + \tfrac{1}{2}(k_1 + k_2)$$
$$k_1 = hf(x_i, y_i)$$
$$k_2 = hf(x_i + h, y_i + k_1)$$

16. Runge Kutta method of order four

$$k_1 = hf(x_i, y_i),$$
$$k_2 = hf\left(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}k_1\right),$$
$$k_3 = hf\left(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}k_2\right),$$
$$k_4 = hf(x_{i+1}, y_i + k_3),$$
$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4).$$

End of Paper.